NUMBER SYSTEM

- **V** – Viniculum
- **B** – Bracket
- **O** – of
- **D** – Division
- **M** – Multiplica-tion
- **A** – Addition
- **S** – Subtrac-tion

Convert the following recurring terms into their corresponding terms ? (a) $27.\overline{17}$ (b) $27.2\overline{17}$ (c) $0.00\overline{17}$

(a) $x = 27.1717---$

$100x = 2717.1717---$

$100x - x = 2717.17 - 27.17$

$99x = 2690$

$$\boxed{x = \frac{2690}{99}}$$

SHORTCUT :-

$x = 27.\overline{17}$

$\frac{p}{q} = \frac{2717-27}{99}$

$\frac{p}{q} = \frac{2690}{99}$

(b) $x = 27.21717---$

$\frac{p}{q} = \frac{27217 - 272}{990}$

$$\boxed{\frac{p}{q} = \frac{26945}{990}}$$

(c) $\frac{p}{q} = \frac{00017-000}{9900}$

$$\boxed{\frac{p}{q} = \frac{17}{9900}}$$

Q

Ans.

$27.\overline{27} \times 33 + 6$

$\dfrac{(2727 - 27)}{99} \times 33 + 6$

$\dfrac{2700}{3} + 6$

$\Rightarrow 906$

Numbers	Freq^n of no.s as Power cycle
0,1,5,6	STAY AS IT IS
2,3,7,8	4
4,9	2

Q What is the unit's digit in the expansion of

Based on cyclicity or Power Cycle

a) $(766)^{136}$

b) $(277)^{134}$

c) $(454)^{41}$

d) $(888)^{103}$

e) $(1028)^{100}$

f) $(459)^{40}$

Ans.

a) $(766)^{136}$

$= 766 \times 766 \times 766 \text{-------} 136$ times

$= \_6 \times \_\_6 \times \_\_6 \text{------}$

$= \_\_\_\_6$

$$\boxed{(\ldots 0/1/5/6)^{\alpha\alpha\alpha \ldots \alpha} = \_\_\_\_\_ 0/1/5/6}$$

b) $(277)^{134} = 277 \times 277 \times 277 \times 277 \text{ ---------- } 134$ times

$= \_\_7 \times \_\_7 \times \_\_7 \times \text{ ---------- } 134$ times

$(\_\_\_\_) \times (\_\_\_) \times (\_\_\_) \times \text{ -------- } \times (277 \times 277)$

49

unit's digit is 9.

$4\overline{)134}$ = 33 ← complete sections

12

14

12

2

SHORTCUT:-

$(277)^{134} \longrightarrow$ Power Cycle is 4

$4\overline{)134}$ = 33

12

14

12

2

$7 \times 7 = 49$

unit's digit

$0^N = 0$

$1^N = 1$

$2^N \downarrow$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

(c) $(454)^{41}$

= Power Cycle = 2

$$2\overline{)41}\;20$$
$$\underline{40}$$
$$1$$

$4^1 = 4$

(d) $(888)^{103}$

Power Cycle is 4

$$4\overline{)103}\;25$$
$$\underline{100}$$
$$3$$

$8^3 = 572$

(e) $(1028)^{100}$

Power cycle is 4

$$4\overline{)100}\;25$$
$$\underline{100}$$
$$0$$

$8^0 = 1$

(f) $(459)^{40}$

Power Cycle is 2

Remainder is 0

$3^N \to 3^1 = 3$
$3^2 = 9$ → Cyclicity is $(3, 9, 7, 1)$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$

$4^N \to 4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$ ← Cyclicity is $(4, 6)$
$4^5 = 1024$

SPECIAL CASE OF REMAINDER ZERO:-

- All Complete sections
- No Incomplete sections

$(1028)^{100} \to$ Power Cycle is 4

$\to 8^4 = 8^2 \times 8^2$
$= 64 \times 64$
$= \ldots 6$

$$4\overline{)100}$$
$$\underline{100}$$
$$0$$

$(459)^{40} \to$ Power Cycle is 2.

$$2\overline{)40}\;20$$
$$\underline{40}$$
$$0$$

$9^2 = 9 \times 9 = 81$

Remainder '0' SHORTCUT:-

After getting Remainder '0'

- Even Base = 6
- Odd Base = 1

Q What is the unit's digit in the expansion of the following expression :- $(666)^{666} \times (877)^{134} + (959)^{20}$

Sol^n

$(666)^{666} \times (877)^{134} + (959)^{20}$

$= (\ldots 6 \times \ldots 9) + \ldots + 1$

$= (\ldots 4) + (\ldots 1) = \ldots 5$

$$4\overline{)134}\;33$$
$$\underline{132}$$
$$2$$

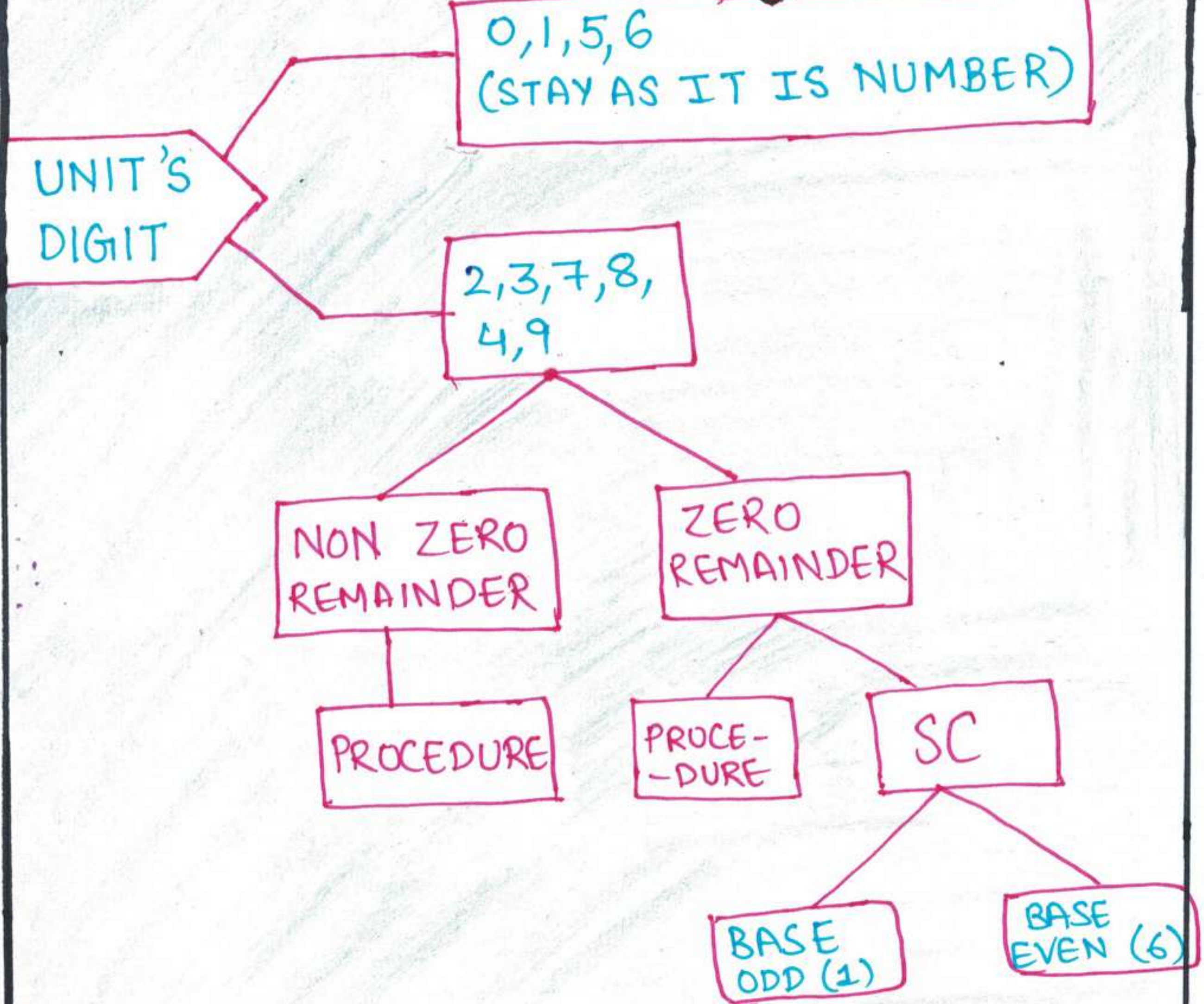
$7^2 = 49$

$$4\overline{)20}\;5$$
$$\underline{20}$$
$$0$$

$(959) \to$ Odd Base

$= 1$

Summary



Q How many zeroes are there at last in the expansion of

- $25 \times 4 \times 8 \times 7 \times 10 \times 16 \times 100$
- $(25)^{125} \times 4^{40}$

Sol^n

$25 \times 4 \times 8 \times 7 \times 10 \times 16 \times 100$

$100 \times 56 \times 16 \times 1000$

$56 \times 16 \times 100000$

= 5 ZEROS

$(25)^{125} \times 4^{40}$

$= (5)^{250} \times (2)^{80}$ ← 80 ZEROS

OTHER METHOD:-

$= 5^2 \times 2^2 \times 2^3 \times 7 \times 5 \times 2 \times 2^4 \times 2^2 \times 5^2$

$= 2^{12} \times 5^5 \times 1$

$= 2^7 \times 2^5 \times 5^5$

$= 2^7 \times 10^5$

= 5 ZEROES

ZERO'S AT LAST CONDITION :-

- Multiple of 10 → direct multiple i.e. 10, 100, 1000 ---
- Hidden multiple → (2, 5)

The total no. of (2×5) combinations = no. of zeroes at last in the expansion.

(total no. of (2×5) comb^n) = (no. of zeros at last in expansion)

Q. How many zeros are there at last in the expansion of :- ☑ 6! ☑ 10! ☑ 100! ☑ 145! ☑ 1000!

Sol^n

☑ $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 6 \times 5^1 \times 3 \times 2^3$

720 = 1 ZERO

☑ $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

3628800 $= 2 \times 5 \times 9 \times 2^3 \times 7 \times 3 \times 2 \times 5 \times 2^2 \times 3 \times 2 \times 1$

$= 2^8 \times 5^2$

= 2 ZEROS

NOTE

1! ; 2! = 2 ; 3! = 6 ; 4! = 24 ; $\boxed{5! = 120}$

↓

only onwards zeros will start coming not before that.

☑ 100! = 100 → 20×5
95 → 19×5
90 → 18×5
85 → 17×5
80 → 16×5
75 → 15×5 = 3×5×5
70 → 14×5
65 → 13×5
60 → 12×5
55 → 11×5
50 → 10×5
45 → 9×5
40 → 8×5
35 → 7×5
30 → 6×5
25 → 5×5
20 → 5×4
15 → 3×5
10 → 2×5
5 → 1×5

NOTE

For 100! zeroes are by default. They will come by default. and no. of zeroes depends on no. of 5's present in it.

$100! = 1 \times 2 \times 3 \times 4 \times 5 \,|\, \times 6 \times 7 \times 8 \times 9 \times 10 \,|\, \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times 17 \times 18 \times 19 \times 20 \,|\, \text{-----} \times \text{---} \times 100$

$\frac{100}{5}$ = 20 sections ← divide the complete 100! in 20 sections.

In these sections some special nos (which contain 2 5's will also be there) such as already taken into account in dividing sections.

☑ $145!$ = $\frac{145}{5} = 29$

*Second line of 5 will be there

*Third line of 5 will be there.

So, No. of Zeros = $\frac{145}{5} + \frac{145}{25} + \frac{145}{125}$

$= 5^{35}$

= 35 Zeros (at last)

☑ $1000!$

No. of Zeros = $\frac{1000}{5} + \frac{1000}{25} + \frac{1000}{125} + \frac{1000}{625}$

$= 5^{249}$

= 249 Zeros (at last)

* 1st line of 5
* 2nd line of 5
* 3rd line of 5
* 4th line of 5

Summary → No. of Zeros at last in $x!$

$$\frac{x}{5} + \frac{x}{5^2} + \frac{x}{5^3} + \text{-----} + \frac{x}{5^n}$$

NOTE :- After decimal values of Quotients are neglected i.e. (TRUNACTION) i.e. INCOMPLETE SECTIONS

Ques What is the highest power of 3 in the expansion of $100!$?

Sol^n

The above conceptual formula is applicable for the purpose of all the PRIME NUMBERS.

So, $\boxed{\frac{x}{3} + \frac{x}{3^2} + \frac{x}{3^3} + \text{---} \frac{x}{3^n}}$; $\boxed{3^n \leq x}$

$\frac{100}{3} + \frac{100}{9} + \frac{100}{27} + \frac{100}{81} = 33 + 11 + 3 + 1 = 48$

$\boxed{\text{Highest Power of 3 is 48}}$

ANALYSIS :-

$15 \rightarrow 3 \times 5$

* In $100!$ we have max^m power of 5 as:-

$\frac{100}{5} + \frac{100}{25} = 24$

* max^m power of 3 as:-

$\frac{100}{3} + \frac{100}{9} + \frac{100}{27} + \frac{100}{81} = 48$

Maximum Power of 15 will be minimum of (24, 48) i.e. 24.

ANALYSIS:-

- $(10!)^{2!}$; $2! = 2$
  $= \frac{10}{5} = 2$ Zeros at last
  Since 2 times, hence --- multiplied 2 times
  = 4 Zeros at last.
- $(10!)^{3!} = \frac{10}{5} = 2$ Zeros at last
  Since 6 times, hence ---- multiplied 6 times
  = 12 zeros at last.
- $(10!)^{2!} \times (10!)^{3!} = \frac{10}{5} \times 2 \times \left(\frac{10}{5} \times 6\right) = 16$ Zeros at last

Ques

$(40!)^{40}$. How many zeros at last are there?

☑ $9 \times 40!$ ☑ $8 \times 40!$ ☑ $8^{40!}$ ☑ $9^{40!}$

Sol^n

$(40!)^{40} \Rightarrow \left(\frac{40}{5} + \frac{40}{25}\right) 40!$ times

$= 9 \times 40!$ times

So, ☑ $9 \times 40!$ zeros at last are there in $(40!)^{40}$.

Ques

How many factors are there for 1800?

Sol^n

$1800 = 2 \times 900$
$= 2 \times 2 \times 450$
$= 2 \times 2 \times 2 \times 225$
$= 2 \times 2 \times 2 \times 3 \times 75$
$= 2 \times 2 \times 2 \times 3 \times 3 \times 25$
$= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$
$= 2^3 \times 3^2 \times 5^2$

← Every composite numbers can be written in the form of PRIME NUMBERS being multiplied together called as PRIME FACTORISATION METHOD

* Write powers Separately
* Add 1 to all of them.
* Multiply them.

$$\begin{array}{ccccc} 3 & & 2 & & 2 \\ \downarrow{+1} & & \downarrow{+1} & & \downarrow{+1} \\ 4 & \times & 3 & \times & 3 \end{array}$$

$= 36$ factors